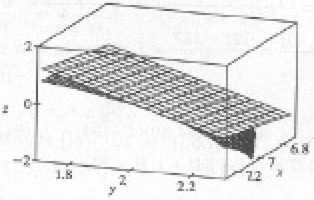
2. $z = f(x, y) = e^{x^2 - y^2} \Rightarrow f_x(x, y) = 2xe^{x^2 - y^2}$, $f_y(x, y) = -2ye^{x^2 - y^2}$, so $f_x(1, -1) = 2$, $f_y(1, -1) = 2$. By Equation 2, an equation of the tangent plane is $z - 1 = f_x(1, -1)(x - 1) + f_y(1, -1)[y - (-1)] \Rightarrow$ $z - 1 = 2(x - 1) + 2(y + 1)$ or $z = 2x + 2y + 1$.

4. $z = f(x, y) = y \ln x \quad \Rightarrow \quad f_x(x, y) = y/x$, $f_y(x, y) = \ln x$, so $f_x(1, 4) = 4$, $f_y(1, 4) = 0$, and an equation of the tangent plane is $z - 0 = f_x(1, 4)(x - 1) + f_y(1, 4)(y - 4) \quad \Rightarrow \quad z = 4(x - 1) + 0(y - 4)$ or $z = 4x - 4$.

14. $f(x,y) = \ln(x - 3y) \quad \Rightarrow \quad f_x(x,y) = \dfrac{1}{x - 3y}$ and $f_y(x,y) = -\dfrac{3}{x - 3y}$, so $f_x(7,2) = 1$ and $f_y(7,2) = -3$. Then the linear approximation of f at $(7,2)$ is given by

$$\begin{aligned} f(x,y) &\approx f(7,2) + f_x(7,2)(x-7) + f_y(7,2)(y-2) \\ &= 0 + 1(x-7) - 3(y-2) = x - 3y - 1 \end{aligned}$$

Thus $f(6.9, 2.06) \approx 6.9 - 3(2.06) - 1 = -0.28$. The graph shows that our approximated value is slightly greater than the actual value.



18. From the table, $f(16,30) = 9$. To estimate $f_T(16,30)$ and $f_v(16,30)$ we follow the procedure used in Section 11.3. Since $f_T(16,30) = \lim_{h \to 0} \dfrac{f(16+h,30) - f(16,30)}{h}$, we approximate this quantity with $h = \pm 4$ and use the values given in the table: $f_T(16,30) \approx \dfrac{f(20,30) - f(16,30)}{4} = \dfrac{14-9}{4} = 1.25$,

$f_T(16,30) \approx \dfrac{f(12,30) - f(16,30)}{-4} = \dfrac{3-9}{-4} = 1.5$.

Averaging these values gives $f_T(16,30) \approx 1.375$. Similarly, $f_v(16,30) = \lim_{h \to 0} \dfrac{f(16,30) - f(16,30)}{h}$,

so we use $h = \pm 10$: $f_v(16,30) \approx \dfrac{f(16,40) - f(16,30)}{10} = \dfrac{7-9}{10} = -0.2$,

$f_v(16,30) \approx \dfrac{f(16,20) - f(16,30)}{-10} = \dfrac{11-9}{-10} = -0.2$. Averaging these values gives $f_v(16,30) \approx -0.2$. The linear approximation, then, is

$$\begin{aligned} f(T,v) &\approx f(16,30) + f_T(16,30)(T-16) + f_v(16,30)(v-30) \\ &\approx 9 + 1.375(T-16) - 0.2(v-30) \end{aligned}$$

Thus when $T = 14$ and $v = 27$, $f(14,27) \approx 9 + 1.375(14-16) - 0.2(27-30) = 6.85$, so we estimate the wind-chill index to be approximately 6.85 °C.

2. First we draw a line passing through Muskegon and Ludington. We approximate the directional derivative at Muskegon in the direction of Ludington by the average rate of change of snowfall between the points where the line intersects the contour lines closest to Muskegon. In the direction of Ludington, the snowfall changes from 60 to 70 inches. We estimate the distance between these two points to be approximately 28 miles, so the rate of change of annual snowfall in the direction given is approximately $\frac{70-60}{28} \approx 0.36$ in/mi. [If we talk of snowfall (rather than annual snowfall), the units are (in/year)/mi.]

6. $f(x,y) = xe^{-2y} \quad \Rightarrow \quad f_x(x,y) = e^{-2y}$ and $f_y(x,y) = -2xe^{-2y}$. If $\mathbf{u}$ is a unit vector in the direction of $\theta = \frac{\pi}{2}$, then $D_{\mathbf{u}} f(5,0) = f_x(5,0)\cos\frac{\pi}{2} + f_y(5,0)\sin\frac{\pi}{2} = 1 \cdot 0 + (-10)1 = -10$.

8. $f(x,y) = y \ln x$

(a) $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle = \langle y/x, \ln x \rangle$ (b) $\nabla f(1,-3) = \langle \frac{-3}{1}, \ln 1 \rangle = \langle -3, 0 \rangle$

(c) By Equation 9, $D_{\mathbf{u}} f(1,-3) = \nabla f(1,-3) \cdot \mathbf{u} = \langle -3, 0 \rangle \cdot \langle -\frac{4}{5}, \frac{3}{5} \rangle = \frac{12}{5}$.

22. $f(x, y, z) = x^2y^3z^4 \quad \Rightarrow \quad \nabla f(x, y, z) = \langle 2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3 \rangle$, $\nabla f(1, 1, 1) = \langle 2, 3, 4 \rangle$. Thus the maximum rate of change is $|\nabla f(1, 1, 1)| = \sqrt{29}$ in the direction $\langle 2, 3, 4 \rangle$.

26. The fisherman is traveling in the direction $\langle -80, -60\rangle$. A unit vector in this direction is $\mathbf{u} = \frac{1}{100}\langle -80, -60\rangle = \langle -\frac{4}{5}, -\frac{3}{5}\rangle$, and if the depth of the lake is given by $f(x, y) = 200 + 0.02x^2 - 0.001y^3$, then $\nabla f(x, y) = \langle 0.04x, -0.003y^2\rangle$. $D_{\mathbf{u}} f(80, 60) = \nabla f(80, 60) \cdot \mathbf{u} = \langle 3.2, -10.8\rangle \cdot \langle -\frac{4}{5}, -\frac{3}{5}\rangle = 3.92$. Since $D_{\mathbf{u}} f(80, 60)$ is positive, the depth of the lake is increasing near $(80, 60)$ in the direction toward the buoy.